Uplift Model Evaluation with Ordinal Dominance Graphs

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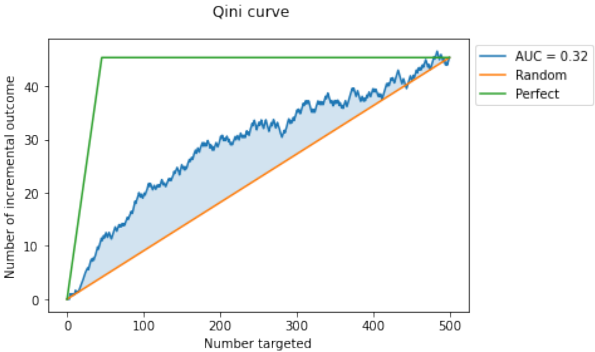
Agenda

1. Qini: How to incorporate information
2. ROCini: How to evaluate
   1. Ordinal Dominance Graphs
   2. Bonus: confidence intervals
3. To evaluate the evaluation metrics

Qini [1]: a recap



$$Q\left(\varphi\right) = \frac{n_T^1(\varphi)}{n_T} - \frac{n_C^1(\varphi)}{n_C}$$

Some remarks about Qini

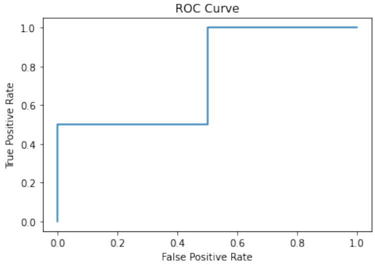
Advantages

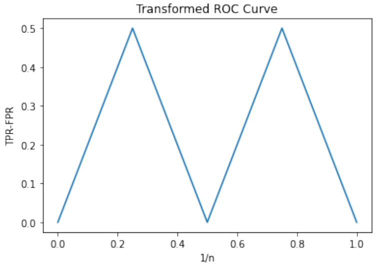
- Visual information
- Intuition: multiplicative and additive counting

Disadvantages

- General inconveniences
- Endpoint is not fixed
- Both terms can compensate for each other ("averaging")
- Confidence intervals: not the most suited object

Back to the basics: ROC and TPR-FPR





Identification: ROC and Qini

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$$\frac{n_T^1(\varphi)}{n_T} \leftrightarrow TPR(\varphi)$$

$$\frac{n_C^1(\varphi)}{n_C} \leftrightarrow FPR(\varphi)$$

ROCini: a different way of measuring

$$Q\left(\varphi\right) = \frac{n_T^1(\varphi)}{n_T} - \frac{n_C^1(\varphi)}{n_C}$$

$$ROCini\left(\varphi\right) = \left(\frac{n_T^1(\varphi)}{n_T^1} - \frac{n_T^0(\varphi)}{n_T^0}\right) + \left(\frac{n_C^0(\varphi)}{n_C^0} - \frac{n_C^1(\varphi)}{n_C^1}\right)$$

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- T: high $\frac{n_T^1(\varphi)}{n_T^1}$, low $\frac{n_T^0(\varphi)}{n_T^0}$
- C: high $\frac{n_C^0(\varphi)}{n_C^0}$, low $\frac{n_C^1(\varphi)}{n_C^1}$

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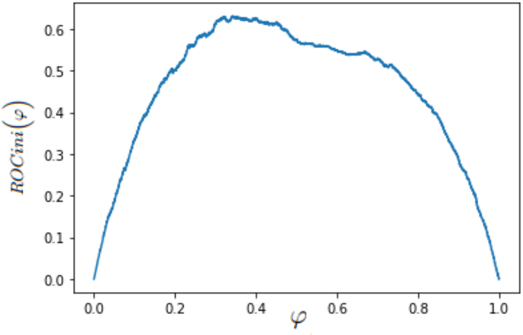
Ranking: T according P_T^1, C according to $1 - P_C^1 \implies$ "average ranking" according to $1 - P_C^1 + P_T^1 = 1 + ITE$

ROCini

Figure: Example of a ROCini curve



Bridge the gap: Ordinal Dominance Graphs

Definition ([2])

For an arbitrary $t \in]-\infty, \infty[$ we can define $G(t)$ as follows:

$$G(t) = \big(X(t), Y(t)\big) = \big(\mathbb{P}(X \leq t), \mathbb{P}(Y \leq t)\big)$$

$ODG(X, Y)$: all points $G(t)$

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Theorem ([2])

The ROC-curve is an ODG with $\mathbb{P}(X \leq t) = FPR(t)$ and $\mathbb{P}(Y \leq t) = TPR(t)$

pROCini: best of both worlds

$$pROCini(X(\varphi), Y(\varphi)) = \left(\frac{\frac{n_T^1(\varphi)}{n_T^1} + \frac{n_C^0(\varphi)}{n_C^0}}{2}, \frac{\frac{n_T^0(\varphi)}{n_T^0} + \frac{n_C^1(\varphi)}{n_C^1}}{2} \right)$$

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Qini-style

$$A(X, Y) = \int (Y(t) - X(t))dt$$

ODG-style

$$A(X, Y) = \int Y(t)X'(t)dt$$

Confidence Intervals

Confidence bounds for AUROC values are suited for ODG! [3, 4, 5]

$$\left[A(X,Y) - z_{\frac{\alpha}{2}} \times Var(A(X,Y))\,,\, A(X,Y) + z_{\frac{\alpha}{2}} \times Var(A(X,Y))\right]$$

Binormal model [2]

$$X \sim N(\mu_X, \sigma_X^2) \quad \& \quad Y \sim N(\mu_Y, \sigma_Y^2)$$
$$Y - X \sim N(\mu_Y - \mu_X, \sigma_X^2 + \sigma_Y^2)$$

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$$Y - X \sim N(\mu_Y - \mu_X, \sigma_X^2 + \sigma_Y^2)$$

$$A(X, Y) = \phi\left(\frac{\mu_Y - \mu_X}{\sqrt{\sigma_Y^2 + \sigma_X^2}}\right)$$

Simulation Study: compare metrics

Algorithm 1 Simulation

```
Input:
   r: the number of runs
   N: the total number of individuals in the population
   α: the expectation of the probabilities for the control group
   v: the variance of the individual uplifts
   ϵ: the variance of the Gaussian error

Initialize:
   probabilities of positive outcome in control group (PC) ∼ N(α, 0.1);
   individual uplifts (IU) ∼ N(0, M);
   individual uplifts with Gaussian error IU + (IUn) ∼ N(0, ϵ)
   Clamp PC: PC ∈ [0, 1]
   Clamp IU: PC+IU ∈ [0, 1]
   Clamp IUn: PC+IUn ∈ [0, 1]

for i = 1 to r do
   for k = 1 to N do B(0.5) (Obs)            ▷ Bernoulli experiment
      if Obs = 0 then do experiment B(PC) (Out)
      else
         do experiment B(PC+IU) (Out)
      end if
   end for
   for metric in [Qini,ROCini,pROCini] do
      metric(Obs, Out, IU)                   ▷ score perfect model
      metric(Obs, Out, IUn)                  ▷ score model with error
   end for
end for
```

Results

ε	**Qini**	**ROCini**	**pROCini**	**p-value**	**McNemar**
0.05	56.00	56.19	56.19	0.26(0.007)	0.25(0.009)
0.1	67.76	69.97	69.97	< 0.0001	< 0.0001
0.15	77.7	82.44	82.45	< 0.0001	< 0.0001
0.20	83.14	88.82	88.82	< 0.0001	< 0.0001
0.25	87.66	93.43	93.42	< 0.0001	< 0.0001
0.30	91.81	96.87	96.88	< 0.0001	< 0.0001

Table: Performance table of the Qini, ROCini and pROCini based on the McNemar test for $r = 100000$, $N = 500$, $\alpha = 0.5$ and $v = 0.2$.

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